

# Mass measurements and superallowed $\beta$ decay

J.C. Hardy <sup>a,\*</sup>, I.S. Towner <sup>a,1</sup>, G. Savard <sup>b,c</sup>

<sup>a</sup> Cyclotron Institute, Texas A & M University, College Station, TX 77843, USA

<sup>b</sup> Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA

<sup>c</sup> Department of Physics, University of Chicago, Chicago, IL 60637, USA

Received 12 October 2005; received in revised form 1 December 2005; accepted 3 December 2005

Available online 14 February 2006

## Abstract

A recent Penning-trap measurement of the masses of  $^{46}\text{V}$  and  $^{46}\text{Ti}$  leads to a  $Q_{\text{EC}}$  value that disagrees significantly with the previously accepted value, and destroys overall consistency among the nine most precisely characterized  $T = 1$  superallowed  $\beta$  emitters. This raises the possibility of a systematic discrepancy between Penning-trap measurements and the reaction-based measurements upon which the  $Q_{\text{EC}}$  values depended in the past. We carefully re-analyze  $(n, \gamma)$  and  $(p, \gamma)$  reaction measurements in the  $24 \leq A \leq 28$  mass region, and compare the results to very precise Penning-trap measurements of the stable nuclei  $^{24}\text{Mg}$ ,  $^{26}\text{Mg}$  and  $^{28}\text{Si}$ . We thus determine upper limits to possible systematic effects in the reaction results, and go on to establish limits for the mass of radioactive  $^{26}\text{Al}$ , to which future on-line Penning-trap measurements can be compared. We stress the urgency of identifying or ruling out possible systematic effects.

© 2006 Elsevier B.V. All rights reserved.

PACS: 27.30.+t; 23.40.-s; 24.80.+y

Keywords: Atomic mass;  $Q$ -value; Systematics

## 1. Introduction

Early in 2005, Hardy and Towner [1] published a complete survey of all half-life, decay-energy and branching-ratio measurements pertaining to 20 superallowed  $0^+ \rightarrow 0^+$  decays. For nine of these, the decay  $\mathcal{F}t$  values were determined with a precision of 0.15% or better. These data, further adjusted for radiative and isospin-symmetry-breaking corrections [2], yielded corrected  $\mathcal{F}t$  values that were all the same to within their derived uncertainties. This is a highly satisfactory situation since it confirms the expectations of the Conserved Vector Current (CVC) hypothesis that the corrected  $\mathcal{F}t$  values for Fermi transitions between states of a particular isospin should all be the same. Indeed, this is a prerequisite if the up-down element,  $V_{ud}$ , of the Cabibbo–Kobayashi–Maskawa (CKM) matrix is to be determined from the average  $\mathcal{F}t$  value. The singular advantage that nuclei have over the neutron or the pion for the determination of  $V_{ud}$  is that there are many examples, currently nine with good

precision, that can be averaged together to reduce the uncertainties while concurrently building confidence that no unforeseen pitfalls are lurking in the data.

Later in 2005, the first Penning-trap mass measurement of one of these nine most precisely determined superallowed transitions was published [3]. Of the nine transitions,  $^{46}\text{V}$  had the largest uncertainty associated with its  $Q$  value and, for this reason, it was selected for the Penning-trap measurement. The result was startling. The mass obtained for the parent,  $^{46}\text{V}$ , and the daughter,  $^{46}\text{Ti}$ , both differed substantially from the adopted values in the 2003 mass tables [4], and the mass difference yielded a decay  $Q$  value nearly three standard deviations away from the average value quoted in the Hardy–Towner compilation [1].

All nine of the prime superallowed transitions feed stable daughter nuclei. In these cases four principal methods have been used in the past to determine their decay  $Q$  values: (a) a  $(p, n)$  threshold measurement, (b) a  $(^3\text{He}, t)$   $Q$ -value measurement, (c) a  $Q$ -value difference measurement (between two superallowed transitions) with  $(^3\text{He}, t)$  reactions on a composite target, and (d) a combination of  $(p, \gamma)$  and  $(n, \gamma)$  measurements. For  $^{46}\text{V}$ , there had only been two previous measurements: a threshold  $(p, n)$  measurement of Squier et al. [5] giving  $Q_{\text{EC}} = 7053.3(18)$  keV and a  $(^3\text{He}, t)$  measurement of Vonach

\* Corresponding author. Fax: +1 979 845 1899.

E-mail address: [hardy@comp.tamu.edu](mailto:hardy@comp.tamu.edu) (J.C. Hardy).

<sup>1</sup> Present address: Department of Physics, Queen's University, Kingston, Ont., Canada K7L 3N6.

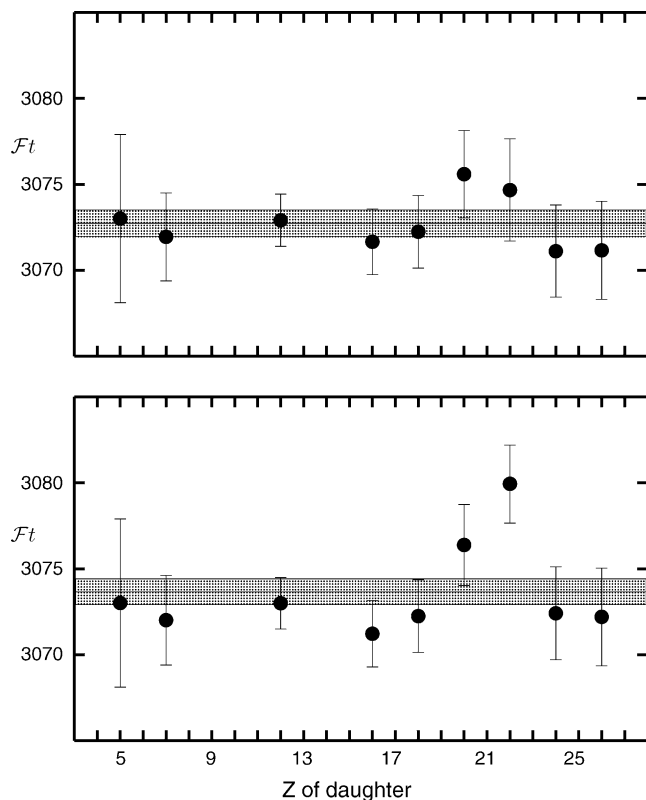


Fig. 1. Upper panel: corrected  $\mathcal{F}t$  values from the Hardy–Towner compilation [1]. Lower panel: revised  $\mathcal{F}t$  values after removing the ( ${}^3\text{He}, t$ ) measurements of Vonach et al. [6] and adding the Penning-trap measurement of Savard et al. [3].

et al. [6] giving  $Q_{\text{EC}} = 7050.41(60)$  keV. The new Penning-trap result [3] of  $7052.90(40)$  keV agrees with the former but is in strong disagreement with the latter, which claims smaller error bars. This latter ( ${}^3\text{He}, t$ ) result appeared in a publication [6] that included six other  $Q$  values in the prime series of superallowed emitters, all quoted with similarly small uncertainties. Since the energy calibration used nearly 30 years ago in Ref. [6] cannot be reconstructed and most of its other six results differ from more recent measurements, Savard et al. [3] opted to remove all seven measurements from the high-precision data set. The consequences are dramatic. In the upper panel of Fig. 1 we show the corrected  $\mathcal{F}t$  values as published in the Hardy–Towner compilation [1] where all values are seen to be in accord. In the lower panel, we show the results of removing the ( ${}^3\text{He}, t$ ) measurements of Vonach et al. [6] and including the Penning-trap measurement of Savard et al. [3]. There is serious deterioration. The data are no longer all consistent as required by the CVC hypothesis: The  $\mathcal{F}t$  value for  ${}^{46}\text{V}$  is significantly above the average.

From a purely experimental perspective, the new Penning-trap result for the  ${}^{46}\text{V}$   $Q_{\text{EC}}$  value disagrees only with a single previous experiment [6], one whose results for other nuclei also appear to be aberrant. Nevertheless, the  $\mathcal{F}t$ -value anomaly it creates clearly raises suspicion and emphasizes the critical importance of extending these Penning-trap measurements to other cases. Suspicion is now further aroused by a preliminary result [7] for the mass of  ${}^{42}\text{Sc}$ , which may also lead to a  $Q_{\text{EC}}$  value that

is higher than the previous average from reaction measurements [1]. At issue now is the question of whether  $Q$  values measured in Penning-traps are consistently higher than those determined by other methods, or whether  ${}^{46}\text{V}$  is an isolated occurrence. If the latter is the case, then the restoration of CVC consistency would seem to depend on there being a deficiency in the current nuclear-structure dependent corrections [2] obtained for  ${}^{46}\text{V}$ . If the former is the case, then all  $Q$ -value measurements, past and present, need to be scrutinized even more carefully for undetected systematic effects.

In this paper, we critically examine two key reactions used in the past to measure the  $Q$  values of superallowed transitions. We compare ( $p, \gamma$ ) and ( $n, \gamma$ ) reactions in the  $24 \leq A \leq 28$  mass region with very precise (conventional) Penning-trap measurements of stable nuclei, and set limits on possible undetected systematic effects in these reactions. We then derive a very conservative range for the mass excess of radioactive  ${}^{26}\text{Al}$  based on these reactions alone. When  ${}^{26}\text{Al}$  is measured by (on-line) Penning-trap, it should provide a critical test for the presence of systematic effects. If the Penning-trap result does not lie within our range, then considerable urgency will have to be placed on resolving the discrepancy and possibly completely re-evaluating all the accepted superallowed  $Q$  values.

## 2. A test of consistency

The masses of  ${}^{24}\text{Mg}$ ,  ${}^{26}\text{Mg}$  [8] and  ${}^{28}\text{Si}$  [9] have all been measured with Penning-trap mass spectrometers. These same masses can also be interconnected by reaction  $Q$  values, the masses of  ${}^{24}\text{Mg}$  and  ${}^{26}\text{Mg}$  being related by a pair of ( $n, \gamma$ ) reactions, on  ${}^{24}\text{Mg}$  and  ${}^{25}\text{Mg}$ ; and the masses of  ${}^{26}\text{Mg}$  and  ${}^{28}\text{Si}$  being related by a pair of ( $p, \gamma$ ) reactions, on  ${}^{26}\text{Mg}$  and  ${}^{27}\text{Al}$ . This provides a so-far unique situation in which Penning-trap and reaction measurements can be directly compared at relatively high precision. The reaction results are quoted to a precision of 80–300 eV and the Penning-trap results are quoted to 2–32 eV.

Since the trap and reaction techniques are very different, they should not share any common sources of systematic error. Therefore, by combining the two ( $n, \gamma$ ) reaction  $Q$  values and comparing the result with the ( ${}^{26}\text{Mg}-{}^{24}\text{Mg}$ ) mass difference obtained from the Penning-trap measurements, we can have some measure of systematic errors associated with that reaction; and similarly by combining the two ( $p, \gamma$ ) reaction  $Q$  values and comparing the result with the ( ${}^{28}\text{Si}-{}^{26}\text{Mg}$ ) mass difference we can determine possible systematic errors in ( $p, \gamma$ ) measurements. The outcome will allow us to judge the validity of reaction  $Q$ -value results in general, but particularly among short-lived exotic nuclei, for which Penning-trap measurements are unavailable or are known with less precision.

### 2.1. ( $n, \gamma$ ) Reactions

In the first row of Table 1 we give the mass excesses of  ${}^{24}\text{Mg}$  and  ${}^{26}\text{Mg}$  as measured with a Penning-trap [8]. There have been five independent measurements of the  $Q$  values for ( $n, \gamma$ ) reactions on the three stable isotopes of magnesium [10–14]. We have taken the  ${}^{24}\text{Mg}(n, \gamma){}^{25}\text{Mg}$  and  ${}^{25}\text{Mg}(n, \gamma){}^{26}\text{Mg}$   $Q$  values

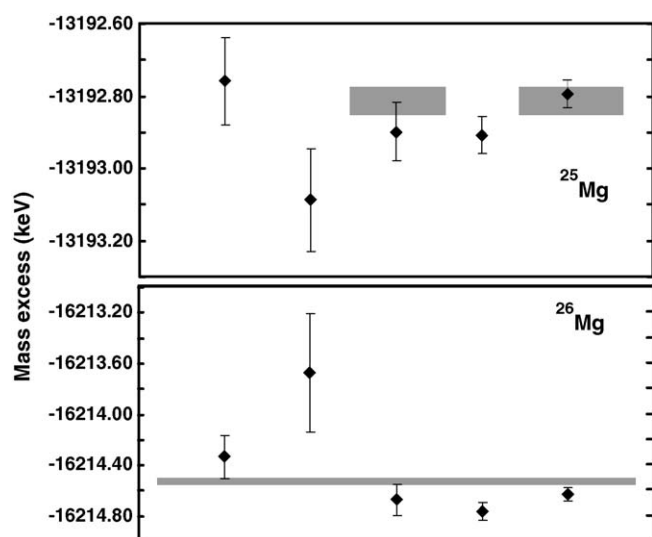


Fig. 2. The mass excesses of  $^{25}\text{Mg}$  and  $^{26}\text{Mg}$  as determined from the Penning-trap-measured mass excess [8] of  $^{24}\text{Mg}$  with the inclusion of the  $^{24}\text{Mg}(n, \gamma)$  reaction (for  $^{25}\text{Mg}$ ) and the  $(n, \gamma)$  reactions on both  $^{24}\text{Mg}$  and  $^{25}\text{Mg}$  (for  $^{26}\text{Mg}$ ). In each panel the five data points correspond to references [10–14] (left to right). The shaded band in the bottom panel is the experimental mass excess of  $^{26}\text{Mg}$  as determined by the Penning-trap measurement [8]. The broken band in the top panel is the average of only the two  $^{25}\text{Mg}$  data points [12,14] through which it passes.

as obtained in each measurement and combined them with the  $^{24}\text{Mg}$  mass excess from the first row of the table to derive a mass excess for  $^{26}\text{Mg}$ . These five results appear in rows 2 through 6 of the last column, where they can each be compared with the higher precision  $^{26}\text{Mg}$  mass excess from the top row. This comparison is also illustrated in the bottom panel of Fig 2 where the  $(n, \gamma)$  results appear as solid diamonds with error bars, and the Penning-trap result is presented as a shaded band. It can be seen that two of the  $(n, \gamma)$  data points for  $^{26}\text{Mg}$  differ significantly from the Penning-trap value, one of them by three standard deviations.

We have examined the original  $(n, \gamma)$  publications [10–14] in detail in order to determine, if possible, how the  $\gamma$ -ray energies used in their calibration have changed in the interim since the experiments were originally analyzed. Three of the references [10,12,13] state that their calibrations relied importantly on  $\gamma$  rays observed from the  $^{14}\text{N}(n, \gamma)^{15}\text{N}$  reaction, some of whose energies depended on the  $Q$  value contemporaneously accepted for that reaction. As can be seen in Table 2, there have been significant changes in the  $^{14}\text{N}(n, \gamma)^{15}\text{N}$   $Q$  value over the period of time involved [4,15–18]. Nevertheless, one of the three mag-

Table 2

The  $Q$  values for the  $^{14}\text{N}(n, \gamma)^{15}\text{N}$  reaction as accepted at various dates in the past 30 years

Date	$Q$ value (keV)	$\Delta$ (keV)	Ref
2003	10833.2961(11)	0	[4]
1993	10833.234(11)	-0.062	[15]
1990	10833.232(20)	-0.064	[16]
1983	10833.302(12)	+0.006	[17]
1977	10833.361(60)	+0.065	[18]

The value  $\Delta$  gives the difference between that  $Q$  value and the one accepted in 2003.

nesium  $(n, \gamma)$  measurements [12] occurred at a time when that  $Q$  value [17] coincidentally agreed with its modern value. It is noteworthy that this measurement produced a  $^{26}\text{Mg}$  mass excess that agrees well with the Penning-trap value (see Table 2). For the other two measurements [10,13] based in part on the  $^{14}\text{N}$   $Q$  value, the fact that only some of their calibration  $\gamma$  rays will have been affected by changes in that  $Q$  value makes it impossible for us to adjust with any confidence the original magnesium  $Q$  values to correct for these changes. In fact, any simple scaling of the derived  $Q$  values by the ratio of the contemporary  $^{14}\text{N}$   $Q$  value to the presently accepted value leads to even larger discrepancies between these results and the Penning-trap value for  $^{26}\text{Mg}$ .

Of the two remaining measurements, one [11] is calibrated relative to the  $\gamma$  rays from  $^{28}\text{Al}$  and  $^{36}\text{Cl}$ , neither of which offer any better opportunities to update the original results from 1982. The other is the most recent [14] and is as yet unpublished; it is calibrated against several sources of delayed and prompt  $\gamma$  rays. Though the actual energies used are not specified, we presume that they are up to date and need no adjustment. This result also is within 100 eV of the Penning-trap value for  $^{26}\text{Mg}$ —although still slightly outside the very tight uncertainties quoted.

This leaves two  $(n, \gamma)$  results [12,14] that remain valid by modern standards and, from Table 1 and Fig. 2, we can see that both results lie slightly below the Penning-trap result, by 150(130) and 100(60) eV respectively. Keeping in mind that each quoted  $(n, \gamma)$  result actually represents the successive application of two such measurements—on  $^{24}\text{Mg}$  and  $^{25}\text{Mg}$ —we can certainly conclude that any systematic differences between Penning-trap and individual  $(n, \gamma)$  measurements must be less than 100 eV. We may also suggest, if not conclude, that uncertainties originally quoted on  $(n, \gamma)$  results may have been assigned with some degree of optimism.

In general, if  $(n, \gamma)$  measurements are to be used in the determination of precise masses, then it is essential that the calibration energies originally used be examined carefully. If they

Table 1

Mass excesses (in keV) of  $^{24}\text{Mg}$  and  $^{26}\text{Mg}$  as measured by Penning-trap; and the mass excesses of  $^{25}\text{Mg}$  and  $^{26}\text{Mg}$  as determined from the  $^{24}\text{Mg}$  mass excess by addition of the measured  $Q$  values for the  $^{24}\text{Mg}(n, \gamma)$  and  $^{25}\text{Mg}(n, \gamma)$  reactions taken from the referenced measurements

Method	Ref.	$^{24}\text{Mg}$	$^{25}\text{Mg}$	$^{26}\text{Mg}$
Penning-trap	[8]	-13933.576(13)		-16214.529(32)
$(n, \gamma)$	[10]		-13192.760(120)	-16214.340(170)
	[11]		-13193.090(140)	-16213.680(460)
	[12]		-13192.900(80)	-16214.680(120)
	[13]		-13192.910(50)	-16214.770(70)
	[14]		-13192.790(40)	-16214.630(50)

Table 3

Mass excess (in keV) of  $^{28}\text{Si}$  as measured by a Penning-trap, and as determined from the  $^{26}\text{Mg}$  mass (see Table 1) by addition of the measured  $Q$  values for the  $^{26}\text{Mg}(p, \gamma)$  and  $^{27}\text{Al}(p, \gamma)$  reactions. The process used to correct the  $(p, \gamma)$  result is described in the text

Penning-trap [9]	$(p, \gamma)$ [19,20]	
	Uncorrected	Corrected
−21492.7968(19)	−21492.090(640)	−21492.590(360)

differ significantly from modern more-precise standards, then there is no simple way that the results can be updated. If they are to be used at all in their uncorrected form, the quoted uncertainties must be increased by at least the amount of the change in the calibration  $Q$  value and probably by more. Finally, it must be added that our conclusions here are drawn from careful measurements that were performed under clean conditions; not all  $(n, \gamma)$  measurements can necessarily be presumed to have such small systematic uncertainties.

## 2.2. $(p, \gamma)$ Reactions

The mass excesses of  $^{26}\text{Mg}$  and  $^{28}\text{Si}$  have both been measured with a Penning-trap [8,9]. The result for the former was listed in Table 1; the latter appears in the first column of Table 3. These two nuclei are also related by the  $Q$  values for the reactions  $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$  and  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$ . In contrast though to the situation with the  $(n, \gamma)$   $Q$  values discussed in the previous section, there has only been one measurement of each reaction  $Q$  value—Refs. [19,20], respectively—published within the past 40 years. The result of our combining the published  $Q$  values from these two measurements with the mass excess of  $^{26}\text{Mg}$  to obtain the  $^{28}\text{Si}$  mass excess is given in the second column of the table, which is labeled “uncorrected”. This result differs from the Penning-trap value by 710 eV, slightly more than the combined uncertainties.

Since both these  $(p, \gamma)$  measurements took place in 1978, we again examined the original publications in detail to establish what calibrations were used and how they have changed in the intervening 27 years. For the  $(p, \gamma)$   $Q$ -value measurements it turned out to be possible to up-date the original results with reasonable confidence and, in fact, to reduce the uncertainties originally quoted. We deal first with the proton resonance energies and then with the energies of the emitted  $\gamma$  rays.

In both measurements, the proton resonance energies were calibrated relative to a particular  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$  resonance that had been established in 1970 to be at  $E_p = 991.880(40)$  keV by an absolute velocity technique [21]. This value has been revised since, with Wapstra [22] in 2003 recommending the value 991.830(50) keV although his recommendation apparently did not include provision for the very precise remeasurement, 991.724(21) keV, made by Brindhaban and Barker [23] 10 years earlier. We choose a conservative approach and adopt the value 991.780(60) keV, which encompasses both these recent values. This amounts to a 100-eV shift in the proton-energy standard from the time the  $(p, \gamma)$  experiments [19,20] were calibrated. When converted to the center-of-mass system, this change increases the  $Q$  values quoted in

both experiments by 96 eV, and marginally increases their uncertainties.

The basis for  $\gamma$ -ray energy calibration was also common to both experiments and depended on the energies of  $\gamma$  rays from the decay of  $^{66}\text{Ga}$  as given by Heath in 1974 [24]. These  $^{66}\text{Ga}$   $\gamma$ -ray energies have been reviewed again quite recently by Helmer and van der Leun [25]: their recommended energies are systematically shifted from the Heath values—though generally not outside of Heath’s quoted error bars—and their uncertainties are significantly smaller.

The  $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$   $Q$  value measured in Ref. [19] utilized two pairs of  $\gamma$  rays in  $^{27}\text{Al}$ , all of energy  $\sim 4.5$  MeV. A comparison of the modern  $^{66}\text{Ga}$   $\gamma$ -ray energies with the Heath catalog values used in the original experiment indicates that the energies in this region have increased by approximately 70 eV. The reaction  $Q$  value originally quoted in the paper must therefore be increased by 140 eV to account for this calibration shift. At the same time, the quoted uncertainty, which was dominated by Heath’s uncertainties in the calibration energies, can be reduced significantly. Incorporating the shifts both in the resonance proton energy and in the calibration  $\gamma$  rays, we revise the  $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$   $Q$  value, which was originally quoted [19] as 8271.0(5), to 8271.2(3) keV.

The paper reporting the  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$   $Q$  value [20] specifically correlated the  $\gamma$  rays in  $^{28}\text{Si}$  that were used to measure the  $Q$  value with the  $\gamma$  rays from  $^{66}\text{Ga}$  that were used to calibrate them. We could then correct each one for the known shift between the  $^{66}\text{Ga}$  energies originally used [24] and their modern counterparts [25]. This has the effect of increasing the originally quoted  $Q$  value by 220 eV from this cause alone and, once again, decreases its uncertainties. Incorporating the shifts both in the resonance proton energy and in the calibration  $\gamma$  rays, we revise the  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$   $Q$  value, which was originally quoted [20] as 11584.5(4), to 11584.8(2) keV.

Combining the corrected values just obtained for the two  $(p, \gamma)$   $Q$  values with the Penning-trap result for the mass excess of  $^{26}\text{Mg}$ , we obtain the “corrected” result for  $^{28}\text{Si}$  shown in the third column of Table 3. It agrees well with the Penning-trap value, differing by only 210 eV, well under its 360-eV uncertainty. On this basis, and again remembering that two  $(p, \gamma)$  measurements were applied to obtain the  $^{28}\text{Si}$  mass, we can conclude that any systematic differences between Penning-trap and  $(p, \gamma)$  results must be less than 200 eV. As with the  $(n, \gamma)$  measurements, here too reliable calibration has proved all important but, at least in the  $(p, \gamma)$  experiments we considered, the original calibration methods were transparent enough that corrections could be applied to adjust the results to modern standards. Once again, though, we must caution that not all  $(p, \gamma)$  measurements can necessarily be presumed to have such small systematic uncertainties.

## 2.3. Test outcome

Our object in this consistency test was to search for possible systematic differences between modern, highly precise Penning-trap measurements of nuclear masses and the results of reaction measurements that have been frequently used in the past

to determine mass differences. Until recently, such reaction  $Q$ -value measurements were the only precise way to obtain the masses of radioactive nuclei and, in cases where precision really mattered, uncertainties as low as 120 eV have been quoted in published measurements. In the recent survey of superallowed  $\beta$  decay [1], the  $Q_{EC}$  values for the nine most precisely known transitions all depended on such reaction  $Q$  values. Any discovery of pervasive systematic effects could have a significant impact on the weak-interaction tests that have been based on superallowed data.

From the test cases we have studied, we can conclude that, if the calibration standards used in the original measurements can be established and if they are consistent with modern values for those standards, then  $(n, \gamma)$   $Q$  values can be considered potentially reliable down to uncertainties of, say, 100 eV. For  $(p, \gamma)$   $Q$  values, the equivalent limit should be around 200 eV. If the original energy-calibration standards are found to have changed significantly, then the two reactions should be dealt with very differently: on the one hand, the  $(n, \gamma)$   $Q$  values cannot easily be updated and they should either be discarded entirely or have their uncertainties increased by at least the amount of the calibration change; on the other hand, the  $(p, \gamma)$   $Q$  values can be updated reliably if the changes in their calibration energies can be clearly documented.

### 3. Evaluation of the $^{26}\text{Al}$ mass

With guidelines now determined for the use of  $(n, \gamma)$  and  $(p, \gamma)$   $Q$ -value measurements, we can turn to establishing the most reliable reaction-based value for the  $^{26}\text{Al}$  mass excess. This is an important issue since the  $Q_{EC}$  for the superallowed  $\beta$  transition between  $^{26}\text{Al}$  and  $^{26}\text{Mg}$  is a key component of stringent weak-interaction tests [1] and, as yet, its mass has not been measured with a Penning-trap. Because of the tight restraints we have just been able to place on the reaction measurements in this mass region,  $^{26}\text{Al}$  should also become a valuable test of Penning-trap measurements on radioactive species. The three Penning-trap measurements we considered in Section 2 were all of stable nuclei,  $^{24}\text{Mg}$ ,  $^{26}\text{Mg}$  and  $^{28}\text{Si}$ , and were conducted under ideal conditions: their uncertainties were only a few eV. Short-lived radioactive nuclei demand a more complex experimental arrangement on-line to an accelerator, and furthermore the collected ions can only be retained in the measurement trap for a relatively short time, thus limiting precision. To date, such measurements quote uncertainties of a few hundred eV and, in fact, none has yet been confronted by results of comparable precision based on a different technique. Of course, it is also true that most reaction measurements have not been confronted by such an independent check at this level of precision either.

Our first step is to determine the mass excess of  $^{25}\text{Mg}$ . In Table 1, we show the result of taking the  $Q$  value for the  $^{24}\text{Mg}(n, \gamma)^{25}\text{Mg}$  reaction from Refs. [10–14] and combining them with the Penning-trap mass excess for  $^{24}\text{Mg}$ . These values are also plotted in the top panel of Fig. 2, where they show considerable scatter. However, in Section 2.1 we determined that only two sets of  $(n, \gamma)$  measurements [12,14] were consistent with modern calibration standards and yielded results for the

Table 4  
Input data for our determination of the mass excess of  $^{26}\text{Al}$

Description	Data (keV)
$^{25}\text{Mg}$ mass excess from $(n, \gamma)$	
Average, published values [12,14]	−13192.812(37)
Adjusted for possible systematics	−13192.752(100)
$^{25}\text{Mg}(p, \gamma)$ $Q$ value	
As published [26]	6306.400(60)
Corrected for calibration changes	6306.426(100)
Adjusted for possible systematics	6306.426(200)
$^{26}\text{Al}$ mass excess	
Not adjusted for possible systematics	−12210.27(11)
Adjusted for possible systematics	−12210.21(22)

$^{26}\text{Mg}$  mass that were within 150 eV of the Penning-trap result. Consequently, we accept only the values for the  $^{25}\text{Mg}$  mass excess associated with those two references and average the results to obtain the value −13192.812(37) keV. This average is shown as a (broken) shaded band in Fig. 2.

There is a further concern. We have noted already that our results for the  $^{26}\text{Mg}$  mass showed that the  $(n, \gamma)$ -based results were 100–150 eV lower (more negative) than the Penning-trap value. Although, this effect was only barely significant statistically, to be safe we choose to increase our derived  $^{25}\text{Mg}$  mass excess by 60 eV and to increase its overall uncertainty to 100 eV to incorporate provision for possible systematic effects. The resultant final value of −13192.752(100) keV also appears in Table 4. Within uncertainties, it agrees with the value −13192.830(30) keV listed in the 2003 Mass Tables [4].

The next step is to examine measurements of the  $^{25}\text{Mg}(p, \gamma)^{26}\text{Al}$   $Q$  value, from which the  $^{26}\text{Al}$  mass can be derived. Although there is only one publication [26] that claims precision below 500 eV, it originates from the same laboratory as, and with one author in common with, the two  $(p, \gamma)$  papers [19,20] we considered in Section 2.2. Even though it was published 13 years later, equivalent calibration adjustments to those made successfully on the earlier results might be expected to achieve similar success with the later one. The uncorrected  $Q$  value appears as it was published in Table 4, and we now deal with what corrections must be applied so that it can be made to conform with modern standards.

The proton resonance energies in the  $^{25}\text{Mg}(p, \gamma)^{26}\text{Al}$  measurement [26] were calibrated relative to the same resonance in the  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$  reaction as were the earlier measurements. However, by the time of the later measurement, the resonance energy was taken to be 991.86(3) keV, 20 eV lower than had been accepted previously. As explained in Section 2.2, more recent evaluations and measurements have led us to adopt the value 991.780(60) keV for the resonance energy and, after conversion to the center-of-mass system, this requires an increase of 76(60) eV in the total  $Q$  value.

The excitation energies in  $^{26}\text{Al}$  that were used in Ref. [26] were taken from earlier work at the same laboratory [27], which measured the energies of  $\gamma$  rays in  $^{26}\text{Al}$  based on those from  $^{66}\text{Ga}$  decay. Unfortunately, the  $^{66}\text{Ga}$   $\gamma$ -ray energies were not listed in Ref. [27] but instead were attributed to Alderliesten et al. [28], whose results did not actually appear in print until 5 years later.

Assuming that the energies of the  $^{66}\text{Ga}$   $\gamma$  rays did not change in the time between their use in calibration and their publication, we can compare the Alderliesten et al. energies with the recent review by Helmer and van der Leun [25]: in the important region around 4 MeV they are, on average, 35 eV higher than the currently accepted values but by a slightly larger amount at higher energies. Because the specific  $\gamma$  rays used in calibration are impossible to ascertain at the end of this tortuous path, we choose to reduce the  $Q$  value by 50(50) eV, a conservative approach.

Applying these two corrections, which tend to cancel one another, we obtain the corrected value for the  $^{25}\text{Mg}(p, \gamma)^{26}\text{Al}$   $Q$  value of 6306.426(98) keV, where the uncertainty simply includes the original experimental uncertainty and the uncertainties we have assigned to the calibration corrections. In Section 2.2, we concluded that we could not rule out systematic uncertainties in  $(p, \gamma)$   $Q$ -value measurements below the level of 200 eV; so, once again to be safe, we increase the overall uncertainty on our final value to 200 eV to accommodate possible undetected systematic effects. Both our calibration-corrected and final results for the  $Q$  value are also listed in Table 4.

Finally, we combine our values for the  $^{25}\text{Mg}$  mass excess and the  $^{25}\text{Mg}(p, \gamma)^{26}\text{Al}$   $Q$  value to obtain a value for the  $^{26}\text{Al}$  mass excess. We do this first with the results that are not adjusted for possible systematic effects: i.e. we use the first number listed for the  $^{25}\text{Mg}$  mass excess in Table 4 and the second number listed for the  $(p, \gamma)$   $Q$  value. The result is  $-12210.27(11)$  keV. Next, using the numbers that take account of possible systematic effects, we obtain the result  $-12210.21(22)$  keV. Both results compare favorably with the value  $-12210.31(6)$  keV listed in the 2003 Mass Tables [4] but, because we incorporate updated calibration standards and, for our second result, include provisions for possible systematic effects, our uncertainties are considerably larger.

One note of caution should be added. Although the mass of  $^{25}\text{Mg}$  is derived from two independent but concordant  $(n, \gamma)$  measurements that also correctly obtain the mass excess for  $^{26}\text{Mg}$ , the link between  $^{25}\text{Mg}$  and  $^{26}\text{Al}$  depends on a single  $(p, \gamma)$  measurement and must be regarded as less secure. Although that measurement came from the same laboratory, and used the same calibrations as  $(p, \gamma)$  measurements that we have demonstrated to agree with precise Penning-trap results on stable nuclei, no corroborating  $(p, \gamma)$  measurements on  $^{25}\text{Mg}$  exist. Obviously, we cannot rule out an experimental aberration.

#### 4. Conclusions

We have carefully analysed measurements of  $(n, \gamma)$  and  $(p, \gamma)$   $Q$  values in the  $24 \leq A \leq 28$  mass region and, after updating the results (where possible) to modern calibration standards, compared them with very precise Penning-trap mass measurements of stable isotopes. This has allowed us to set upper limits on possible systematic effects of 100 eV for  $(n, \gamma)$  reactions and of 200 eV for  $(p, \gamma)$  reactions. Based on  $(n, \gamma)$  and  $(p, \gamma)$  reactions, we then established two values for the mass excess of radioactive  $^{26}\text{Al}$ , one excluding and the other including the adjustment

and limits we obtained for possible unobserved systematic contributions.

Neither result should be regarded as a value we recommend for use in determining the superallowed transition energy from  $^{26}\text{Al}$ . Instead, together they provide a critical standard for reaction-based results, to which a future on-line Penning-trap mass measurement can be compared. If the Penning-trap result turns out to lie within the limits of our first value (the one uncorrected for possible systematic effects), then one can be reasonably confident that actual systematic effects are below the upper limits we set; in that case Penning-trap measurements, when they proliferate, can simply be averaged in with the earlier reaction-based results. If the Penning-trap result lies outside the limits of our first value but inside the limits of our second value (adjusted for systematics), then one must suspect that reaction measurements in general may suffer from undiagnosed systematic effects; wherever their quoted uncertainties are in the few-hundred-eV region, they will need to be increased accordingly.

If the Penning-trap result lies outside the range of even our systematics-adjusted result, then that could be a sign of serious systematic difficulties, which could call into question all reaction-based measurements of superallowed transition energies or, conversely, could cast doubt on the precision of on-line Penning-trap measurements of radioactive isotopes. This would require serious and urgent attention, particularly in the evaluation of superallowed  $\beta$  decay and its associated weak-interaction tests.

On-line Penning-trap measurements of  $^{26}\text{Al}$  (and  $^{25}\text{Mg}$ ) to perform the comparison test are strongly recommended.

#### Acknowledgements

This work was supported by the U.S. Department of Energy under Grant No. DE-FG03-93ER40773 and by the Robert A. Welch Foundation under Grant No. A-1397.

#### References

- [1] J.C. Hardy, I.S. Towner, Phys. Rev. C 71 (2005) 055501.
- [2] I.S. Towner, J.C. Hardy, Phys. Rev. C 66 (2002) 035501.
- [3] G. Savard, F. Buchinger, J.A. Clark, J.E. Crawford, S. Gulick, J.C. Hardy, A.A. Hecht, J.K.P. Lee, A.F. Levand, N.D. Scielzo, H. Sharma, K.S. Sharma, I. Tanihata, A.C.C. Villari, Y. Wang, Phys. Rev. Lett. 95 (2005) 102501.
- [4] G. Audi, A.H. Wapstra, C. Thibault, Nucl. Phys. A 729 (2003) 337.
- [5] G.T.A. Squier, W.E. Burcham, S.D. Hoath, J.M. Freeman, P.H. Barker, R.J. Petty, Phys. Lett. B 65 (1976) 122.
- [6] H. Vonach, P. Glaessel, E. Huenges, P. Maier-Komor, H. Roesler, H.J. Scheerer, H. Paul, D. Semrad, Nucl. Phys. A 278 (1977) 189.
- [7] G. Savard et al., to be published.
- [8] I. Bergström, M. Björkhage, K. Blaum, H. Bluhme, T. Fritioff, Sz. Nagy, R. Schuch, Eur. Phys. J. D 22 (2003) 41.
- [9] F. DiFilippo, V. Natarajan, K.R. Boyce, D.E. Pritchard, Phys. Rev. Lett. 73 (1994) 1481.
- [10] M.A. Islam, T.J. Kennett, S.A. Kerr, W.V. Prestwich, Can. J. Phys. 58 (1980) 168.
- [11] P. Hungerford, H.H. Schmidt, Nucl. Instrum. Methods 192 (1982) 609.
- [12] W.V. Prestwich, T.J. Kennett, Can. J. Phys. 68 (1990) 261; W.V. Prestwich, T.J. Kennett, Can. J. Phys. 68 (1990) 1352 (erratum).

- [13] T.A. Walkiewicz, S. Raman, E.T. Jurney, J.W. Starnier, J.E. Lynn, *Phys. Rev. C* 45 (1992) 1597.
- [14] H.D. Choi, R.B. Firestone, R.M. Lindstrom, G.L. Molnar, S.M. Mughabghab, R. Paviotti-Corcuera, Z. Revay, A. Trkov, V. Zerkin, C.M. Zhou, Database of prompt gamma Rays from Slow Neutron Capture for Elemental Analysis, International Atomic Energy Agency, to be published.
- [15] G. Audi, A.H. Wapstra, *Nucl. Phys. A* 565 (1993) 1.
- [16] A.H. Wapstra, *Nucl. Instrum. Methods* 292 (1990) 671.
- [17] E.R. Cohen, A.H. Wapstra, *Nucl. Instrum. Methods* 211 (1983) 153.
- [18] A.H. Wapstra, K. Bos, *Atomic Data Nucl. Data Tables* 19 (1977) 177.
- [19] J.W. Maas, A.J.C.D. Holvast, A. Baghus, H.J.M. Aarts, P.M. Endt, *Nucl. Phys. A* 301 (1978) 237.
- [20] J.W. Maas, E. Somorjai, H.D. Graber, C.A. van den Wijngaart, C. van der Leun, P.M. Endt, *Nucl. Phys. A* 301 (1978) 237.
- [21] M.L. Roush, L.A. West, J.B. Marion, *Nucl. Phys. A* 147 (1970) 235.
- [22] A.H. Wapstra, G. Audi, C. Thibault, *Nucl. Phys. A* 729 (2003) 129.
- [23] S.A. Brindhaban, P.H. Barker, M.J. Keeling, W.B. Wood, *Nucl. Instrum. Methods A* 340 (1994) 436.
- [24] R.L. Heath, *Gamma-ray Spectrum Catalog*, Aerojet Nuclear Company, 1974.
- [25] R.G. Helmer, C. van der Leun, *Nucl. Instrum. Methods A* 450 (2000) 450.
- [26] S.W. Kikstra, Z. Guo, C. Van der Leun, P.M. Endt, S. Raman, T.A. Walkiewicz, J.W. Starnier, E.T. Jurney, I.S. Towner, *Nucl. Phys. A* 529 (1991) 39.
- [27] P.M. Endt, P. de Wit, C. Alderliesten, *Nucl. Phys. A* 459 (1986) 61; P.M. Endt, P. de Wit, C. Alderliesten, *Nucl. Phys. A* 476 (1988) 333.
- [28] C. Alderliesten, J.A. van Nie, A.P. Slok, P.M. Endt, *Nucl. Instrum Methods A* 335 (1993) 219.